Complex Permittivity Measurements of Common Plastics Over Variable Temperatures

Bill Riddle, James Baker-Jarvis, *Senior Member, IEEE*, and Jerzy Krupka, *Senior Member, IEEE*

***Abstract*—In this paper, we present complex permittivity data at microwave frequencies (approximately 10 GHz) for many common plastics over a temperature range of 122 to 375 K. The measurements were made with a $\mathrm{TE}_{01\delta}$ dielectric resonator placed inside an environmental chamber. Data are presented for the following materials: acrylonitrile butadiene styrene, polytetrafluoroethylene, cross-linked polystyrene, tetrafluorethylene-perfluorpropylene, polypropylene, polysulfone, polymethylmethacrylate, polyvinyl chloride, polycarbonate, high-density polyethylene, polyoxy-methylene (acetal homopolymer), and polyamide.**

***Index Terms*—Complex permittivity, dielectric resonator, microwave freqeuncy, plastics, variable temperatures.**

I. INTRODUCTION

PLASTICS ARE used in many areas of manufacturing and have naturally found application in the construction of electronic devices. As the operating frequencies of these devices continue to increase, designers need accurate data of the electromagnetic properties of these materials. There is a sizable body of data in the technical literature about the electromagnetic properties of plastics, but most of those results focus on only a few materials. Permittivity measurements at various temperatures and frequencies for some of the materials presented here can be found in [1]–[9]. An excellent review article presenting data on many plastics with extensive references is given in [10]. Data for several plastic materials using different techniques are presented in [11] and [12].

In this paper, we present complex permittivity data of many common plastics at a frequency of approximately 10 GHz and over the temperature range of 122 to 375 K. The plastics measured are as follows: polytetrafluoroethylene (PTFE), cross-linked polystyrene (CPS), tetrafluorethylene-perfluorpropylene (FEP), polypropylene, polysulfone, polymethylmethacrylate (PMMA), polyvinyl chloride (PVC), polyoxy-methylene (acetal homopolymer), polycarbonate, high-density polyethylene (HDPE), polyamide (nylon), and acrylonitrile butadiene styrene (ABS).[1]

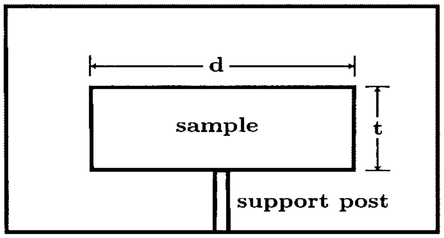

Fig. 1. Cross section of the $\mathrm{TE}_{01\delta}$ dielectric resonator.

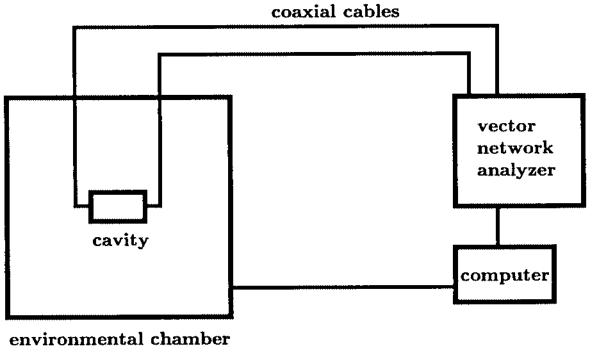

Fig. 2. Schematic diagram of the measurement system.

II. DESCRIPTION OF THE MEASUREMENT SYSTEM

In our previous paper [13], we described the use of a cylindrical $\mathrm{TE}_{01\delta}$ dielectric resonator for measuring the complex permittivity of cylindrical samples of dielectric material over a variable temperature range. A cross-sectional diagram of the resonator geometry with a typical dielectric sample supported by a dielectric post is shown in Fig. 1. The cylindrical cavity is constructed out of metal, and the fields within the sample are excited by coupling loops (not shown) that can be varied to control the amount of electromagnetic energy coupled into the sample.

Fig. 2 shows a schematic diagram of the measurement system with the $\mathrm{TE}_{01\delta}$ cavity placed inside an environmental chamber and connected to a vector network analyzer (VNA) through coaxial bulkhead connectors. At each temperature, the dielectric sample was allowed to reach thermal equilibrium by maintaining a constant chamber temperature for one hour. The magnitude of the transmission parameter S_{21} was measured with the VNA to obtain the resonant frequency and quality factor Q of the resonator using the 3-dB method [14]. Effort was made to maintain the cavity coupling below -45 dB to minimize systematic effects due to the coupling loops. The procedure was automated by use of a computer to control the system.

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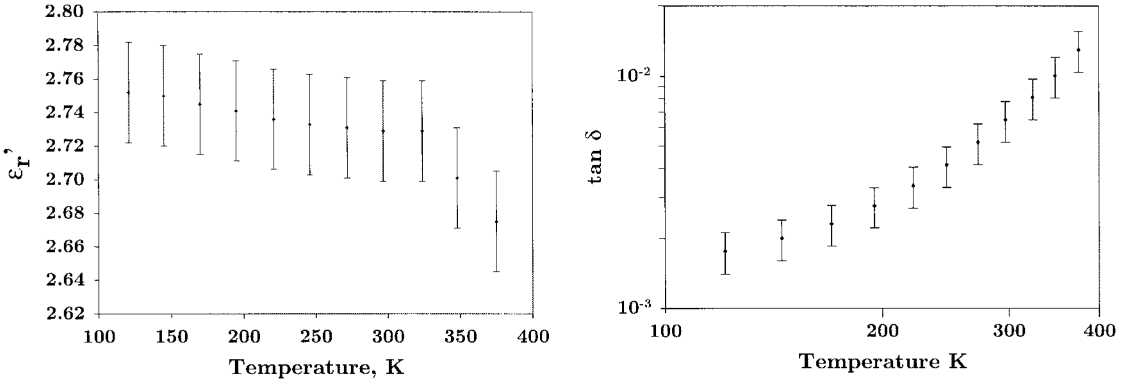
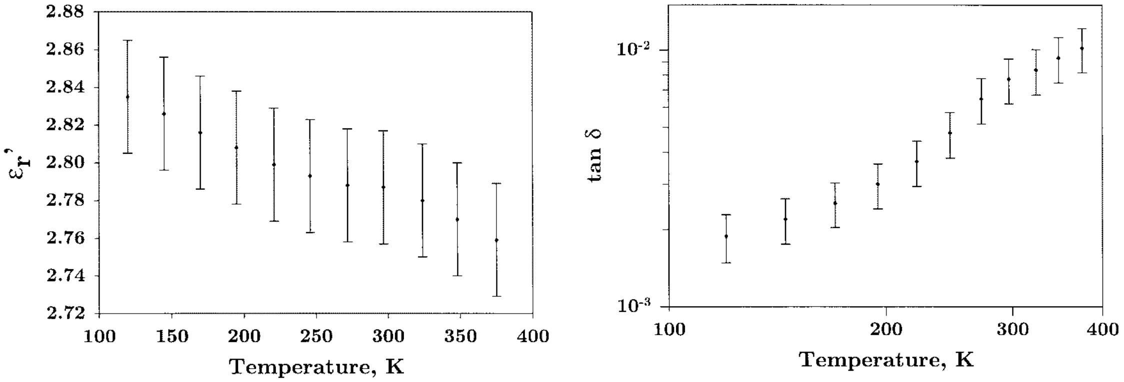
[1] Specific materials that are commonly used are mentioned for informational purposes only. This does not imply or constitute any endorsement by the National Institute of Standards and Technology.



(a) (b)

Fig. 3. Acetal homopolymer, $f \approx 11$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 4. Acrylonitrile butadiene styrene, $f \approx 11$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.

For a dielectric material that is homogeneous, linear, and isotropic, the complex permittivity ϵ is defined by

$$\mathbf{D}(\omega) = \epsilon(\omega)\mathbf{E}(\omega) \tag{1}$$

where $\mathbf{E}$ is the complex time–harmonic steady-state electric field, $\mathbf{D}$ is the corresponding electric flux density, and ω is the radian frequency [15]. For complex time–harmonic fields, it is useful to express ϵ in terms of real and imaginary components

$$\epsilon(\omega) = \epsilon_0(\epsilon_r'(\omega) - j\epsilon_r''(\omega)) \tag{2}$$

where ϵ_0 is the permittivity of vacuum. The measurements in this paper are reported in terms of the relative permittivity ϵ_r' and the dielectric loss tangent, defined as

$$\tan\delta(\omega) = \frac{\epsilon_r''(\omega)}{\epsilon_r'(\omega)}. \tag{3}$$

After measuring the resonant frequency and Q at each temperature for the materials under test, we were able to calculate ϵ based on the algorithm described in [13].

III. THERMAL EXPANSION

The algorithm used in calculating ϵ is very sensitive to variations in sample dimensions, which in turn are functions of temperature. Therefore, it is important to take into account the effect of thermal expansion on the samples during a measurement. In general, for a given material, the coefficient of linear thermal expansion α at constant pressure P is defined by [16]

$$\alpha = \frac{1}{l}\left(\frac{\partial l}{\partial T}\right)_P \tag{4}$$

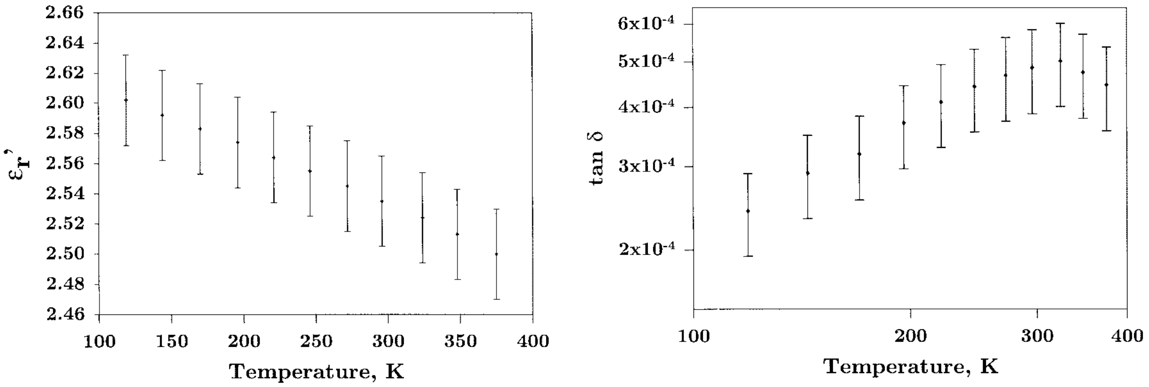
where T is the temperature and l is the linear dimension of interest. Assuming that l is a function only of T, the resulting separable ordinary differential equation can be solved as

$$l_1 = l_0 \exp\left(\int_{T_0}^{T_1} \alpha dT\right) \tag{5}$$

where l_0 and l_1 are the linear dimensions at temperatures T_0 and T_1, respectively. Data for the thermal expansion coefficient α are available for many of the materials measured in this work [16], [17]. Using the data for α with the expression for l_1 above, we were able to correct for the expansion of the measured samples due to changing temperature.

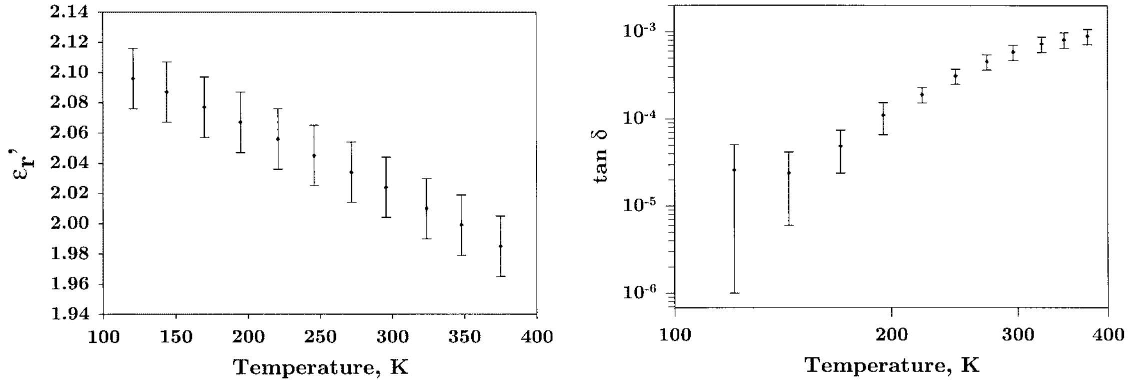
IV. MEASUREMENT RESULTS

The results of the temperature-variable measurements of the plastic materials are shown in Figs. 3–15. Each figure caption includes the approximate resonant frequency of the cavity containing the sample at ambient temperature. Of course, this frequency varies as the properties and dimensions of the sample and cavity change with temperature. The samples measured were machined from common industrial-grade sources, with no



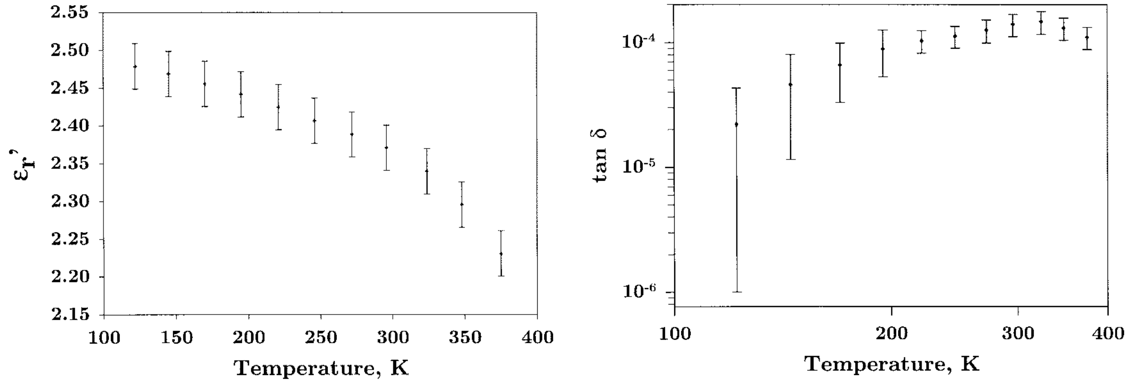
(a) (b)

Fig. 5. Cross-linked polystyrene, $f \approx 10.7$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 6. Tetrafluorethylene-perfluorpropylene, $f \approx 10.4$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 7. High-density polyethylene, $f \approx 11.3$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.

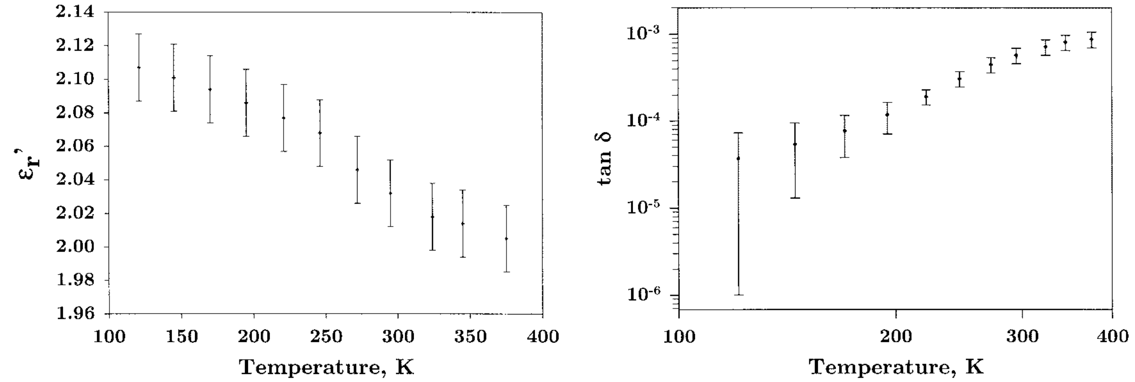
precise information about their exact chemical composition. A common characteristic of plastics is that the composition, and therefore the electrical properties, can vary considerably between different manufacturers and sometimes within different production lots of the same manufacturer. We have not made any effort to analyze or control this possible systematic effect on the data presented here.

V. UNCERTAINTY ANALYSIS

A detailed explanation of the procedure we used to evaluate the standard uncertainty of our measurements can be found in [18]. The standard uncertainty of a measurement is denoted by $u(x_i)$, where x_i is the best estimate of the input quantity of the measurand of interest. All input quantities are assumed to be independent, although in general this assumption is very weak. The combined standard uncertainty of a measurand θ is defined as

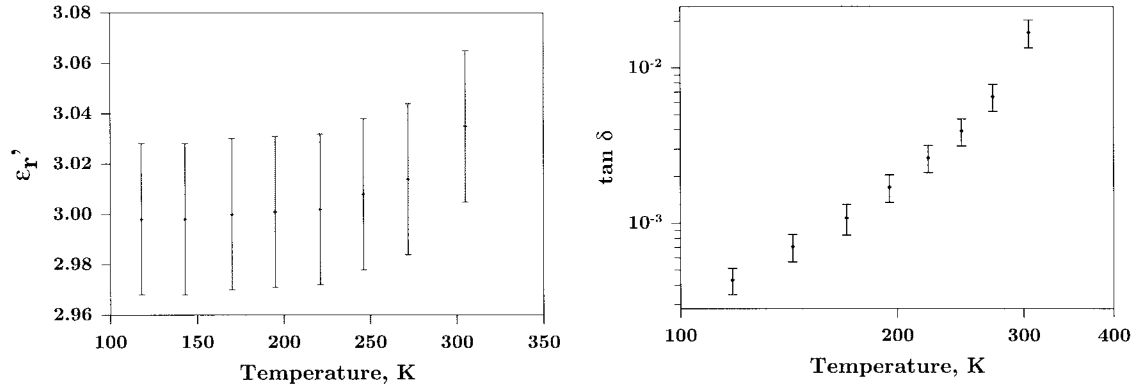
$$u_c(\theta) = \sqrt{\sum_i \left(u(x_i) \frac{\partial \theta}{\partial x_i} \right)^2}. \tag{6}$$

The two measurands in this paper are $\epsilon_r{}'$ and $\tan \delta$. In general, uncertainties that can be evaluated by statistical methods are



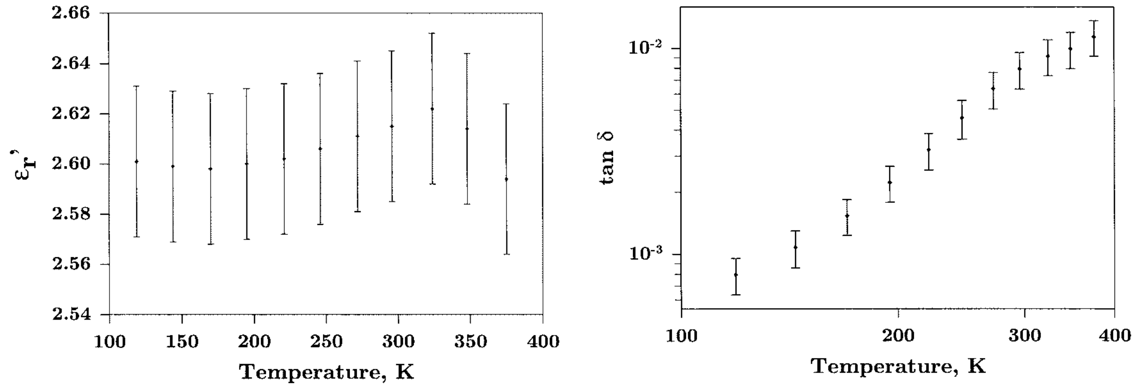
(a) (b)

Fig. 8. High-density polytetrafluoroethylene, $f \approx 11.5$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 9. Polyamide, $f \approx 10.8$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 10. Polymethylmethacrylate, $f \approx 11$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.

termed Type A (reported with degrees of freedom ν), and all others as Type B. The reported uncertainties of the results shown in Figs. 3–15 are given in terms of the expanded uncertainty of a measurand θ, defined as [18]

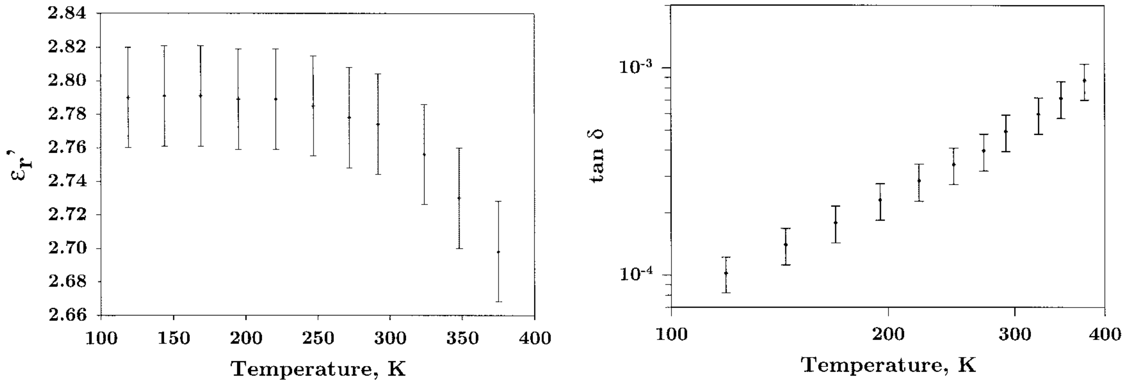
$$U(\theta) = ku_c(\theta) \tag{7}$$

where k is the coverage factor. We have used a value of $k = 2$ for $U(\epsilon_r{}')$ and $U(\tan\delta)$.

Performing the uncertainty calculations on every sample at every temperature proved to be impractical, so instead extensive calculations of the measurement uncertainties were performed on two representative materials (PTFE and CPS). Although we have attempted to include the dominant sources of measurement uncertainty in these calculations, some sources (e.g., temperature effects on the coaxial cables) could not be included.

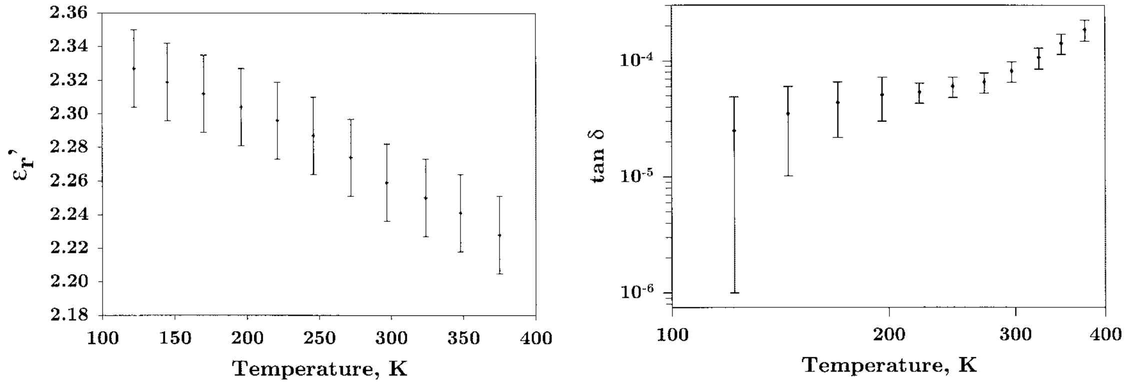
The input quantities that significantly contribute to the combined standard uncertainty of $\epsilon_r{}'$ are the sample thickness t, the sample diameter d (see Fig. 1), and the cavity calibration found by use of a reference material, denoted q. This functional relationship can be expressed as

$$\epsilon_r' = f(t, d, q). \tag{8}$$



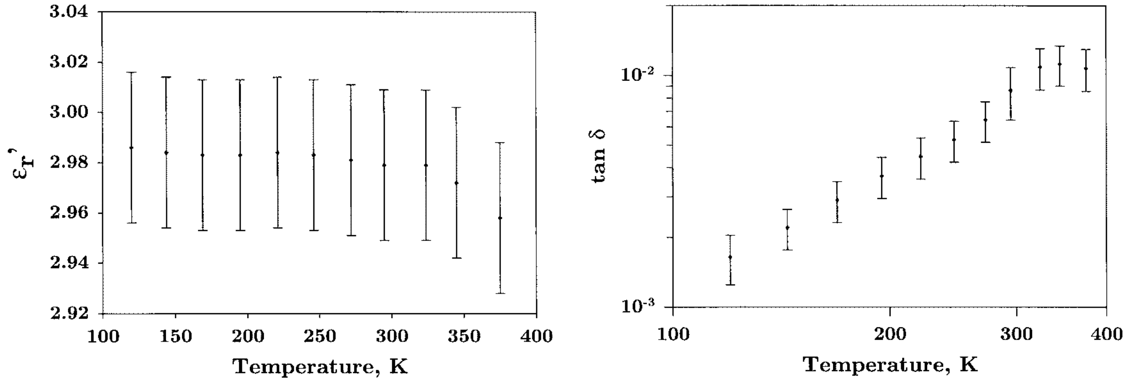
(a) (b)

Fig. 11. Polycarbonate, $f \approx 11$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 12. Polypropylene, $f \approx 9.4$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

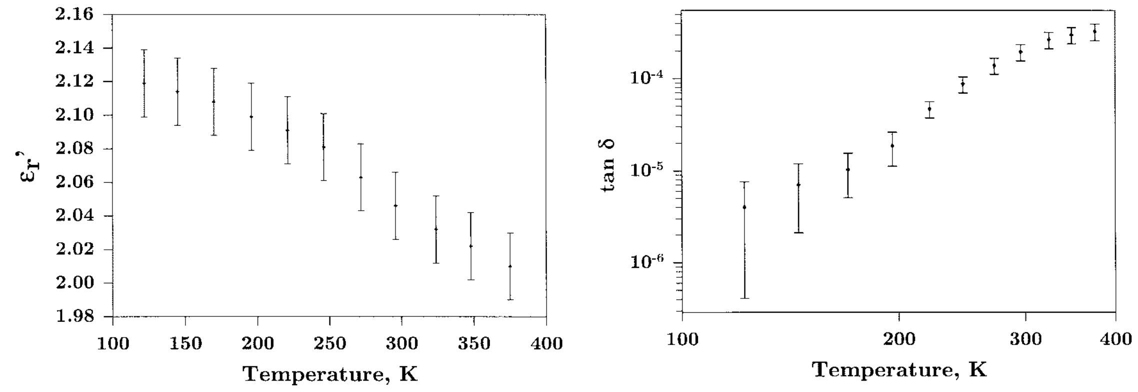
Fig. 13. Polysulfone, $f \approx 10.8$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.

The function f is defined through the numerical algorithms described in [13]. The dimensional uncertainties $u(t)$ and $u(d)$ were obtained from repeated measurements of the sample at ambient temperature and pressure (297 K and 84 kPa). A well-characterized reference material (quartz) was used to calibrate the systematic effect due to the cavity post height, and $u(q)$ was obtained from the uncertainty in the value of ϵ of the reference material.

Listed in Table I are the values used in calculating the combined standard uncertainty for ϵ_r'. The partial derivatives were obtained by numerically varying the input quantity of interest by a small amount and noting the effect in ϵ_r'. By use of the values in Table I, the combined relative uncertainty of ϵ_r' was found to be

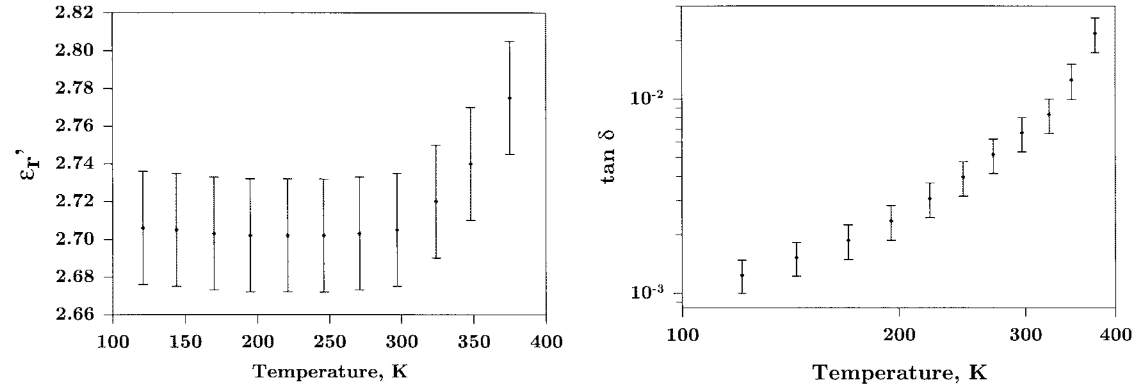
$$\frac{u_c(\epsilon_r')}{\epsilon_r'} = \pm 0.5\%. \tag{9}$$

The uncertainty in the measurement of $\tan\delta$ was dominated by the Type B measurement uncertainties of Q and R_s, the surface resistance of the metal cavity walls. For measurements performed at ambient temperature, the contributions for each of these factors is shown in Table II [13]. In order to determine the effect of $u(Q)$ and $u(R_s)$ on $u_c(\tan\delta)$, we performed a sensitivity analysis on measurement data instead of using



(a) (b)

Fig. 14. Polytetrafluoroethylene, $f \approx 10$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.



(a) (b)

Fig. 15. Polyvinylchloride, $f \approx 11$ GHz. (a) Relative permittivity versus temperature. (b) Loss tangent versus temperature.

TABLE I
RELATIVE UNCERTAINTY COMPONENTS FOR ϵ_r'

Component	Method of Evaluation	Value %
$u(t)(\partial\epsilon_r'/\partial t)$	*Type A*,$\nu = 29$	0.1
$u(d)(\partial\epsilon_r'/\partial d)$	*Type A*,$\nu = 29$	0.1
$u(q)(\partial\epsilon_r'/\partial q)$	*Type B*	0.5

TABLE II
RELATIVE UNCERTAINTY COMPONENTS FOR $\tan\delta$

Component	Method of Evaluation	Value %
$u(Q)(\partial\tan\delta/\partial Q)$	*Type B*	2
$u(R_s)(\partial\tan\delta/\partial R_s)$	*Type B*	10

TABLE III
RELATIVE COMBINED UNCERTAINTY FOR $\tan\delta$ OF LOW-LOSS MATERIALS

Temperature, K	Value %
122	±65
145	±35
170	±25
196	±20
221	±10
246	±10
272	±10
296	±10
324	±10
348	±10
375	±10

direct calculations based on (6). This was necessary because of the way the numerical model accessed values for R_s. For most of the materials, the results of these calculations at all temperatures was found to be

$$\frac{u_c(\tan\delta)}{\tan\delta} = \pm 10\%. \tag{10}$$

Howerever, for materials that exhibited very low loss (PTFE, FEP, HDPE, and polypropylene), the contribution from $u(R_s)$ played a much larger role in $u_c(\tan\delta)$, resulting in a nonlinear uncertainty versus temperature, as shown in Table III. Other factors also influenced the measurement of $\tan\delta$, but only for a few of the samples and only at certain temperatures. These systematic effects were the result of Q resonance asymmetry due to mode distortion and excessive cavity coupling (greater than −45 dB) at lower temperatures. Where appropriate, these sources of uncertainty are included in the data presented in Figs. 3–15 and are estimated based on numerical experiments of the PTFE sample data.

Although the analysis presented here should not be considered exhaustive, it does attempt to reasonably estimate the main sources of measurement uncertainty of the measurands.

VI. CAVEAT

The measurements described in this paper were designed without regard to the temperature limitations of the materials used. Before using any of these materials in any application, the manufacturers data sheets should be consulted to determine the recommended limits of their use.

VII. CONCLUSION

We have presented temperature-dependent data of the complex permittivity of many common plastics at microwave frequencies. The results were presented with an uncertainty analysis based on measurements and experiments of selected representative materials.

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